Modelisation of dense avalanche[1]

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Introduction

An avalanche is a gravitational flow of snow; this definition matches quite a wide variety of snow flows, ranging from the dense snow flow (density over 200 kg/m^3) to the powder snow avalanche which is similar to a turbidity current (like a gravity current) which follows different flow laws.

Out in nature, under our temperate climate, dense snow flows are more likely. However, it is hard to distinguish between a dust avalanche and a dense avalanche: a ground avalanche (heavy snow) could develop into an aerosol whose contribution to the avalanche dynamic will remain negligible. It is only in the case of a fresh and cold snow on a steep slope that a blast also called "aerosol" develops, made of great vertical structures which rapidly separate from the flow: it is the dramatic powder snow avalanche which is modeled in a channel by a heavy fluid flow into a lighter fluid (that is where the analogy with turbidities comes from). These avalanches require very special conditions in order to develop (meteorology, topography, etc.) That is why they remain less frequent than dense avalanches. According to reports, the meteorological scenarios that provoke heavy snow avalanches are of several types:

- huge snow falls;
- milder snow weather with or without rain.

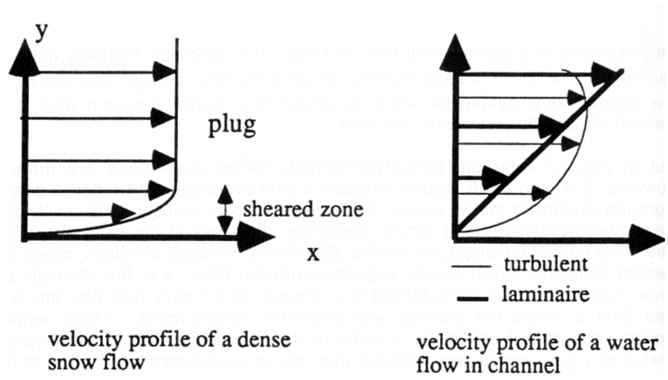
In the following pages, we will focus on dense snow flows, whose modelisation is inspired from fluvial hydraulics and based on shallow water equations in which the rheological law is chosen as Bingham's type. The notion of dense flow can be extended to any avalanche that does not develop any aerosol or to which aerosol makes little or no contribution to the movement of it.

Physical model

Two important remarks lead to the construction of a rheological model for snow. These are:

- *The notion of critical depth*: given a slope and a snow cohesion, the natural release of the avalanche will happen only if the snow depth is greater than a given depth called the critical depth
- *Plug flow*: velocity profile measurements that were taken in laboratory by K. Nishimura and N. Naemo in 1989 showed that the snow fall on a slope is made of a very thin zone of great shear and a no-shear zone called a plug.

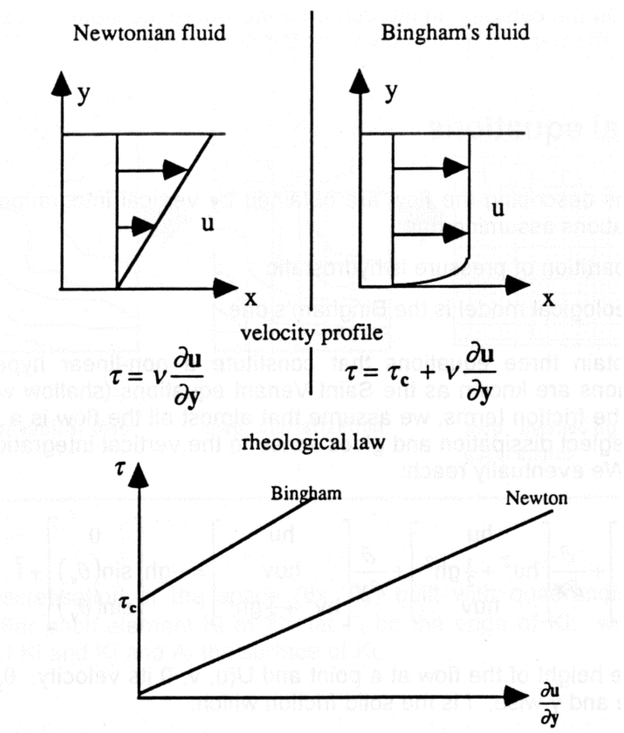
[1] C. Ancey and M. Naaim: "Modelisation of dense avalanches," Comptes Rendus de l'Université d'été, Chamonix 1992 (ANENA, Grenoble, 1995) pp.173–182.



Rheological model

The velocity profile and the existence of critical depth can be understood as characteristics of a threshold fluid (Herschel-Bulkley Law) and, more precisely, of a Bingham fluid. The Bingham model allows shear stress to be connected to local velocity gradient via the relation:

$$\begin{cases} \text{if } \tau_{xy} < \tau_c \text{ then } \dfrac{\partial u}{\partial y} = 0 \\ \text{if } \tau_{xy} > \tau_c \text{ then } \tau_{xy} = \tau_c + \nu \dfrac{\partial u}{\partial y} \end{cases}$$



The flow of such a fluid only happens if the shear stress is greater than τ_c. And in the static snow layer, the stress is given by:

$$\tau = \varrho g h \sin \frac{\partial z_f}{\partial x}$$

Here is only expressed the balance of a block of snow at the height of z_f, undergoing its own weight and solid friction

$$h_c = \frac{\tau_c}{\varrho g \sin \frac{\partial z_f}{\partial x}}$$

This gives the critical depth because there is no way that τ could be greater than τ_c.

Release and snow mechanism

As long as the depth remains smaller than the critical depth, any release is possible. On the other hand, if, thanks to snow fall, to any extra weight or to an increased water content, the shear stress should be greater than the critical stress τ_c, the avalanche could start. On the contrary, in the deposit zone where the slope is less steep, friction disperses the momentum and finally stops the avalanche.

Dynamical equations

The equations describing the flow are obtained by vertical integration of the Navier-Stokes equations assuming that:

- the repartition of pressure is hydrostatic
- the rheological model is the Bingham's one

Thus, we obtain three equations that constitute a non-linear hyperbolic system. These equations are known as the Saint-Venant equations (shallow water). In order to models the friction terms, we assume that almost all the flow is a plug, which will allow us to neglect dissipation and gives sense to the vertical integration we have just calculated. We eventually reach:

$$\frac{\partial}{\partial t}\begin{bmatrix} h \\ hu \\ hv \end{bmatrix} + \frac{\partial}{\partial x}\begin{bmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \\ huv \end{bmatrix} + \frac{\partial}{\partial y}\begin{bmatrix} hv \\ huv \\ hv^2 + \frac{1}{2}gh^2 \end{bmatrix} = -gh\begin{bmatrix} 0 \\ \sin\theta_x \\ \sin\theta_y \end{bmatrix} + \vec{f}$$

where *h* is the height of the flow at a point and *U*(*u*, *v*, 0) its velocity. θ_x and $\sin\theta_y$ are the slopes *x*-wise and *y*-wise. $\vec{f}$ the solid friction which:

- when at a standstill, compensates as much as it can the force of gravity;
- when flowing opposes the velocity;

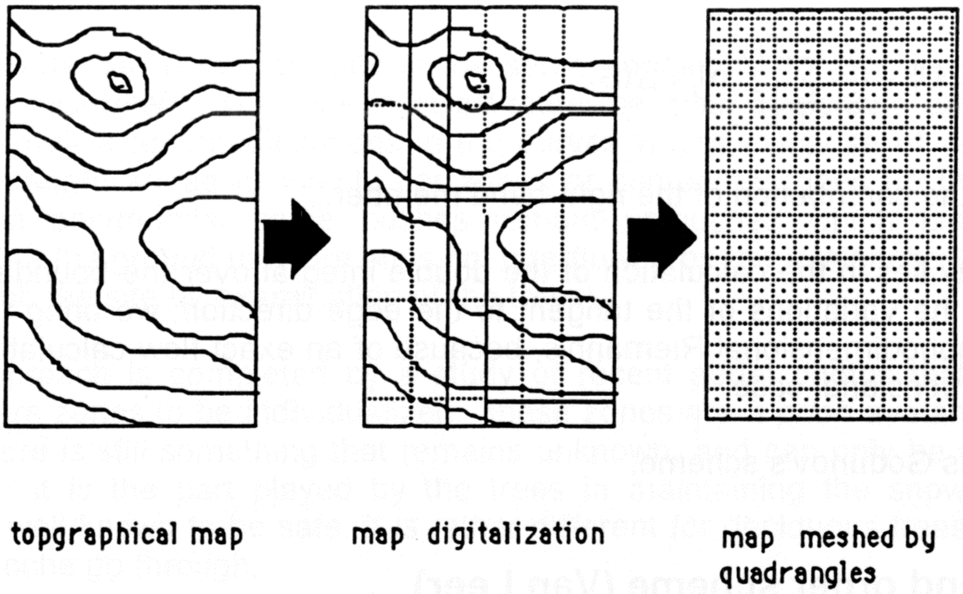
thus friction stress is:

$$\vec{f} = -\frac{\tau_c}{\rho}\sum_{i=1}^{n_i} S_i \frac{(\vec{u} - \vec{u}_i)}{\|\vec{u} - \vec{u}_i\|}$$

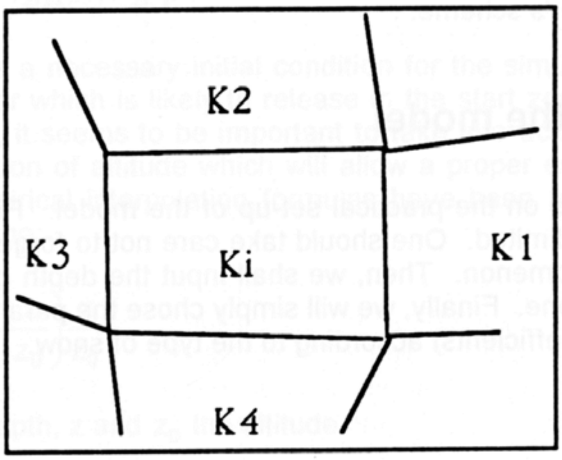
Numerical resolution

We now have a system of equations that governs the motion of the avalanche. ln order to study dense flows in a particular site, we shall compute the topographic map

We now have a system of equations that governs the motion of the avalanche. In order to study dense flows in a particular site, we shall compute the topographic map of the domain as a regular mesh that will cover all of the surface. To achieve that, contour lines will be digitized into a computer, then with an 'interpolation method, the domain will receive a mesh



Let Th be a discretization of the space (Ox, Oy) built with quadrangles K whose diameter is *k*. For each element Ki of Th, let Γi be the edge of Ki; we call Γij the common edge of Ki and Kj and Si the surface of Ki.



The first order scheme (Godunov)

We try to approach *U* = (*h*, *hu*, *hv*) using a distribution constant on each element. We define the spatial average of *U* in the Ki element at t_n and t_{n+1} as the projection of *U*(*x*, *y*, *t*) on Ki. Then, integrating over the volume Ki [t_n , t_{n+1}], we reach the following schema:

$$U_i^{n+1} = U_i^n + \frac{\Delta t}{\mathrm{Si}} \sum_{i=1}^{4} \begin{bmatrix} E \\ F \end{bmatrix} \vec{n} l_i + tS$$

where S is the contribution of the right-hand member.

The difficulty lies in the calculation of the double integral over the boundary of Ki. If we neglect the variations of the tangent to the edge direction, we bring the problem back to a monodimensional Riemann's, because of an exact flow calculation possible in the unidimensional case.

- This is Godunov's scheme

The second order scheme (Van Leer)

A more accurate version is obtained if one considers a linear distribution for each element. This requires two more stages which are the slope prediction stage and the correction stage (in order to avoid the generation of local maxima which would result in oscillations that would make the method unstable).

The so-defined numerical model is stable under the condition of Friedrich-Levy- Current less than 0.6.

- This is Van Leer's scheme.

Application of the model

We are going to focus on the practical set-up of the model. First of all, the domain should be properly delimited. One should take care not to forget any zone that could take part in the phenomenon. Then, we shall input the depth of snow layer that will release in the start zone. Finally, we will simply chose the parameters (critical stress, density, turbulence coefficients) according to the type of snow.

Determining and analyzing the path

To delimitate the work zone, it is useful to have a few notions about the release of dense avalanches. Avalanches release most often when the slope is over 30° and the starting zone is usually a leeward ridge, or at about 50 m downhill of a break in the slope. Localization of these zones thus consists in "reading" the map painting out the zones. In this way, we determine the start lines.

Secondary starts can occur laterally in the same conditions on the slope after the basal layer is disrupted. Then we must estimate the main avalanche paths among the thalwegs with a few modifications in the curves where, because of velocity, the snow goes straight ahead in mainly flat zones or sometimes goes up the hill. As mentioned in documents, some bushes remain untouched which leads to the conclusion that the ground they are on is not overflowed by the avalanche (which is to be checked on the site by examination of the trees).

This first approach is completed by a study of recent aerial photos that allow the steep and bare zones to be individualized: these zones are a priori avalanche paths. However, there is still something that remains unknown, and can only be worked out in the field: it is the part played by the trees in maintaining the snow. Though conifers are well-known to be safe, it is rather different for deciduous trees which will let the avalanche go through.

Finally, any smoothing of the slope should be considered as a potential deposit zone for the avalanche which will spread over by lateral diffusion. The stopping zone is estimated according to the morphology of the site. Generally speaking, there shall be either an important spreading zone on the run-out zone (in this case, it is a cone) or a narrow band on which snow has concentrated as in the case of a gorge opening on a shelf

Definition of the triggering path

The triggering depth is a necessary initial condition for the simulation. We will take the depth of snow Layer which is likely to release in the start zone. ln the case of a steep avalanche gully, it seems to be important to take into account the variation of snow depth as a function of altitude which will allow a proper estimation of the start volume. Several empirical interpolation formulas have been found. For example, E.D.F. uses the following:

$$E(z) = \frac{E(z_0)}{2}\left(1 + \frac{z}{z_0}\right)\frac{z}{z_0}$$

where *E* is the snow depth, *z* and z_0 the altitude.

Adjustment of the coefficients

The model is based upon an initial condition (the triggering depth) or, in the case of an artificial triggering, upon a second parameter (velocity condition). Then, a set of friction coefficients is to be determined – at least two of them are required: the solid friction coefficient and the coefficient of viscous dissipation. Later on, more coefficients can be taken into account in order to express ground conditions, vegetation, etc. The previous coefficients can be modified as a function of the zone the avalanche goes through. But, for the time being, we will only use two coefficients and adjust them according to the type of snow. Thus, the adjustment will take place on small gullies with fairly simple ground.

Application to the Lautaret gully

The Lautaret pass (Hautes Alpes) is an interesting site from the point of view of its excellent snow coverage because of its altitude (2050 m) as well as its easy approach. Many little gullies have been listed and for 20 years information about avalanches has been collected. The measurements concerned the leading edge velocity, the dimensions of the run-out zone, the characteristics of the snow, etc. Progress made possible by image computing has allowed direct knowledge of the velocity evolution at a given point.

One advantage of the Lautaret site is the small dimension of its gullies (500*100 m). The topography is simple (channelled flow, wide run-out zone). According to this known velocity profile, we will fit the coefficient τ_c/ϱ. The run-out zone will allow for adjustment of the coefficient of viscous dissipation. Then, the model can be tested on dense snow avalanches whose characteristics are known (density, humidity, estimation of the critical stress, depth) and the results given by the model are compared to the one *in situ*.

The Lautaret pass is considered to be a classical case allowing for adjustment of the parameters. To validate the model entirely, it will be tested on a gully representative of a classical engineering problem with less complete data. We have chosen the Boulangeard guily (Isère).

Application to the Boulangeard gully

The Boulangeard gully is a more imposing gully because of its scale and its activity regarding avalanches. Two thousand, four hundred meters long, it begins with a little northern cirque above 2000 m and ends in the Eau d'Olle valley at an altitude of 810 m. lt has been studied at length because it directly threatens the reservoir of the Verney dam. An historical study has shown that avalanches could be listed in three main categories:

• The yearly avalanche: it brings nearly 70,000 m^3 of snow and ends up between 800 and 900 m of altitude.

• The avalanche which occurs several times a century: its volume reaches 170,000 m^3 of snow and it stops at around 830 m.

• The major avalanches:the estimated volume exceeds 360,000 m^3 and it spreads down to the valley (at present the retenue of Verney).

The avalanches of the 9th March 1980, the 20th March 1982, and the 4th March 1923 are avalanches representative of each category.

As a result, a rough idea of the precision of the model can be fixed and we can know if the matters of scale can interfere in such a model. The influence that variations in the critical stress and topography and the initial conditions can have is tested at the same time in order to evaluate the versatility of the model and its possible application in the field of engineering.

Conclusion

ln this paper, the main points of the model have been stressed first by displaying assumptions and the method which allowed the equations of motion to be found, then by giving an idea of the method used for the numerical resolution, and finally by giving a survey of the methodology of adjustment and its use in important sites.

This model is still being studied, but present results are very encouraging, especially since the model based on equations of conservation remains very evolutive: a new rheological law, the variation of parameters, will be easily integrated.